PROCEEDINGS of the
23rd International Congress on Acoustics

9 to 13 September 2019 in Aachen, Germany

Simulation of vibrations from railway tunnels

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ABSTRACT

Railway traffic in tunnels leads to annoying vibrations and secondary airborne noise in buildings near the tunnel. A model is presented that introduces a horizontal layering of the anisotropic soil coupled with tunnel shell. The horizontal layering allows us to include reflections of waves at the interfaces. The tunnel shell and the superstructure are simulated by means of finite elements in 2.5D, and the soil is coupled using the boundary element method in 2.5D. The 2.5D model allows us to model a uniformly moving source in a straightforward manner, which is important for high-speed trains. The difficulty of the model is that the Green's function of the layered soil in 2.5D is not given explicitly. This problem is solved by using the solution calculated in the wave-number domain. Therefore, the singular and non-singular boundary element integrals are solved in this domain. An additional numerical step needed is the inverse Fourier transformation of the values integrated over elements.

Keywords: Railway vibration, waves in solids, tunnel

1. INTRODUCTION

Railways in tunnel do not radiate noise directly into the surrounding. Vibrations in the soil, however, can be perceived directly or as secondary air-borne noise in buildings. Therefore, a limitation of such vibrations is needed. Countermeasures against these vibrations are very expensive. The most effective measure is a floating slab construction as the superstructure. A precise modelling of such vibrations is needed in order to plan the mitigation measures exactly where they are needed. Existing models showed differences of up to 10 dB from the measured values (1). One reason is the missing stratification of the soil in the model. In the current article, a new model is presented that allows using a horizontal layering with an anisotropic linear elastic material in every layer as an extension to the approach presented in (2). The material parameters can be defined as complex numbers to add damping to the model.

2. MODEL OF THE SOIL

It is assumed that the soil consists of one isotropic half-space and anisotropic layers above the half-space. The material is assumed to be linearly elastic. This assumption is realistic, because vibrations have only small amplitudes compared to e.g. earthquake events.

2.1 Model of the Anisotropic Layers

The anisotropic material is assumed to have orthogonal material coordinates with respect to the coordinates of the soil. A rotation of the elasticity matrix can be done in tensor description in a simple manner using the cosines between the material coordinates and the global coordinate system.

$$\begin{aligned} \lambda^{im} &= \langle n_i^{(g)}, n_m^{(l)} \rangle \\ E_{ijkl}^{(g)} &= \lambda^{im}\lambda^{jn}\lambda^{ko}\lambda^{lp}E_{mnop}^{(l)}. \end{aligned} \tag{1}$$

2.2 Spectral Description of the Layers

The differential equations of the layers are investigated in a Fourier integral transformed domain (3). The Fourier transformation is done with respect to the longitudinal axis *x,* the transversal coordinate *y,* the vertical coordinate z and time t.

$$\hat{f}\left(k_x,k_y,k_z,\omega\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f\left(x,y,z,t\right)e^{-j\left(k_x x+k_y y+k_z z+\omega t\right)}dx\,dy\,dz\,dt\,. \tag{2}$$

The discretization in the wavenumber frequency domain is given by the wavenumbers k_x, k_y, k_z and the angular frequency ω. The differential equations become algebraic equations in this domain. First, the relation between strains ε and displacements *u* is used:

$$\begin{bmatrix} \hat{\varepsilon}_{xx} \\ \hat{\varepsilon}_{yy} \\ \hat{\varepsilon}_{zz} \\ \hat{\varepsilon}_{xy} \\ \hat{\varepsilon}_{yz} \\ \hat{\varepsilon}_{zx} \end{bmatrix} = \begin{bmatrix} jk_x & 0 & 0 \\ 0 & jk_y & 0 \\ 0 & 0 & jk_z \\ jk_y/2 & jk_x/2 & 0 \\ 0 & jk_z/2 & jk_y/2 \\ jk_z/2 & 0 & jk_x/2 \end{bmatrix} \begin{bmatrix} \hat{u}_x \\ \hat{u}_y \\ \hat{u}_z \end{bmatrix}. \quad (3)$$

In the matrix form, this is written as:

$$\hat{\boldsymbol{\varepsilon}} = \hat{\mathbf{D}}\hat{\mathbf{u}}. \quad (4)$$

The differential matrix **D** becomes in the transformed domain an algebraic matrix depending on the wavenumbers. With the elasticity matrix **E,** a relation between stress **σ** and displacements **u** can be derived, and both together lead to a relation between stresses and displacements:

$$\begin{bmatrix} \hat{\sigma}_{xx} \\ \hat{\sigma}_{yy} \\ \hat{\sigma}_{zz} \\ \hat{\sigma}_{xy} \\ \hat{\sigma}_{yz} \\ \hat{\sigma}_{zx} \end{bmatrix} = \begin{bmatrix} E_{11} & E_{12} & E_{13} & E_{14} & E_{15} & E_{16} \\ E_{12} & E_{22} & E_{23} & E_{24} & E_{25} & E_{26} \\ E_{13} & E_{23} & E_{33} & E_{34} & E_{35} & E_{36} \\ E_{14} & E_{24} & E_{34} & E_{44} & E_{45} & E_{46} \\ E_{15} & E_{25} & E_{35} & E_{45} & E_{55} & E_{56} \\ E_{16} & E_{26} & E_{36} & E_{46} & E_{56} & E_{66} \end{bmatrix} \begin{bmatrix} jk_x & 0 & 0 \\ 0 & jk_y & 0 \\ 0 & 0 & jk_z \\ jk_y/2 & jk_x/2 & 0 \\ 0 & jk_z/2 & jk_y/2 \\ jk_z/2 & 0 & jk_x/2 \end{bmatrix} \begin{bmatrix} \hat{u}_x \\ \hat{u}_y \\ \hat{u}_z \end{bmatrix}. \quad (5)$$

The equation can be written in the matrix-vector form as:

$$\hat{\boldsymbol{\sigma}} = \mathbf{E}\hat{\boldsymbol{\varepsilon}} = \mathbf{E}\hat{\mathbf{D}}\hat{\mathbf{u}}. \quad (6)$$

The balance of the forces gives three additional equations:

$$\begin{bmatrix} jk_x\hat{\sigma}_{xx} + jk_y\hat{\sigma}_{xy} + jk_z\hat{\sigma}_{zx} \\ jk_x\hat{\sigma}_{xy} + jk_y\hat{\sigma}_{yy} + jk_z\hat{\sigma}_{yz} \\ jk_x\hat{\sigma}_{zx} + jk_y\hat{\sigma}_{yz} + jk_z\hat{\sigma}_{zz} \end{bmatrix} + \rho\omega^2 \begin{bmatrix} \hat{u}_x \\ \hat{u}_y \\ \hat{u}_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (7)$$

In this equation, ρ is the mass density and ω the angular frequency. In Eq. 7 it is assumed that no body forces are given inside of the material. Substituting **σ** in Eq. 7 by the relations defined in Eq. 5 gives three equations that depend only on the displacements **u**. The result is a 3x3 matrix that can be seen as a quadratic eigenvalue problem depending on the vertical wave number k_z. Solving the eigenvalue problem leads to eigenvalues $k_{z,i}$ and eigenvectors of the displacements Ψ_i. Using Eq. 6 the eigenvectors of the stresses can be derived from the eigenvectors of the displacements.

A test function is needed in the transformed domain that is non-zero only at the eigenvalues. A Dirac-delta distribution is a good choice:

$$\begin{aligned} \hat{\mathbf{u}}(k_x,k_y,k_z,\omega) &= \sum_{i=1}^{6}\left[A_i\boldsymbol{\psi}_i(k_x,k_y,\omega)\,\delta(k_z-k_{z,i})\right] \\ \hat{\boldsymbol{\sigma}}(k_x,k_y,k_z,\omega) &= \sum_{i=1}^{6}\left[\mathbf{E}\hat{\mathbf{D}}_i A_i\boldsymbol{\psi}_i(k_x,k_y,\omega)\,\delta(k_z-k_{z,i})\right]. \end{aligned} \quad (8)$$

A_i are the unknown amplitudes of the six waves in the medium. The inverse transformation

$$\tilde{f}(k_x,k_y,z,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y,z,t)\,e^{-j(k_x x + k_y y + \omega t)}\,dx\,dy\,dt \quad (9)$$

with respect to the vertical axis z leads to the following equation:

$$\begin{aligned} \tilde{\mathbf{u}}(k_x,k_y,z,\omega) &= \sum_{i=1}^{6}\left[A_i\boldsymbol{\psi}_i(k_x,k_y,\omega)\,e^{jk_{z,i}z}\right] \\ \tilde{\boldsymbol{\sigma}}(k_x,k_y,z,\omega) &= \sum_{i=1}^{6}\left[\mathbf{E}\tilde{\mathbf{D}}_i A_i\boldsymbol{\psi}_i(k_x,k_y,\omega)\,e^{jk_{z,i}z}\right]. \end{aligned} \quad (10)$$

The new matrix **D** changes in that manner that instead of k_z now $k_{z,i}$ is used:

$$\tilde{\mathbf{D}}_i = \begin{bmatrix} jk_x & 0 & 0 \\ 0 & jk_y & 0 \\ 0 & 0 & jk_{z,i} \\ jk_y/2 & jk_x/2 & 0 \\ 0 & jk_{z,i}/2 & jk_y/2 \\ jk_{z,i}/2 & 0 & jk_x/2 \end{bmatrix}. \tag{11}$$

2.3 Horizontally Layered Half-space

Six unknown amplitudes A_i are to be computed in every layer. At the interface between layers u and d three equations for the compatibility of the displacements and three equations for the equilibrium of the stresses are given:

$$\begin{bmatrix} \tilde{\mathbf{u}}_u \\ \tilde{\boldsymbol{\sigma}}_u \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{u}}_d \\ \tilde{\boldsymbol{\sigma}}_d \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \tilde{\mathbf{p}} \end{bmatrix}, \tilde{\mathbf{u}} = \begin{bmatrix} \tilde{u}_x \\ \tilde{u}_y \\ \tilde{u}_z \end{bmatrix} \tilde{\boldsymbol{\sigma}} = \begin{bmatrix} \tilde{\sigma}_{xz} \\ \tilde{\sigma}_{yz} \\ \tilde{\sigma}_{zz} \end{bmatrix}. \tag{12}$$

At the interface external forces **p** are introduced. These forces **p** are the load part in the Green's function. At the free surface, only three equations are given for the stresses, but in the isotropic half-space three waves propagate upwards and three propagate downwards. Setting the upcoming waves to zero fulfills causality and reduces the number of unknowns to three. Therefore, as many unknowns are given as equations for their determination are derived.

2.4 Loading Inside a Layer

For an arbitrary depth d_p of the source term of the Green's function loading inside a layer is needed (4). In Fig. 1, the layer is subdivided into two layers with an additional interface m-n. The upper layer has the unknowns A_i and the lower layer has the unknowns B_i. The thickness of the layer is d.

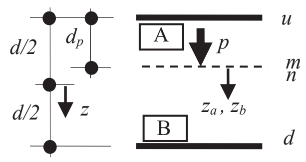

Figure 1 – Division of one layer into two layers with the additional interface m-n at the load depth d_p.

The load is a Dirac delta term in an arbitrary direction.

$$\begin{aligned} p(x,y,z,t) &= \delta(x)\delta(y)\delta(z-z_p)\delta(t) \\ \tilde{p}(k_x,k_y,z,\omega) &= \delta(z-z_p) \qquad , \; z_p = d_p - d/2 . \end{aligned} \tag{13}$$

The stresses and displacements at the four interfaces are given in the transformed domain by

$$\begin{aligned} u:\tilde{\mathbf{u}}_u &= \sum_{i=1}^{6} A_i \boldsymbol{\Psi}_i e^{-jk_{z,i}d_p} \quad , \quad \tilde{\boldsymbol{\sigma}}_u = \sum_{i=1}^{6} A_i \mathbf{F}^{-1}\mathbf{D}\boldsymbol{\Psi}_i e^{-jk_{z,i}d_p} \\ m:\tilde{\mathbf{u}}_m &= \sum_{i=1}^{6} A_i \boldsymbol{\Psi}_i \quad , \quad \tilde{\boldsymbol{\sigma}}_m = \sum_{i=1}^{6} A_i \mathbf{F}^{-1}\mathbf{D}\boldsymbol{\Psi}_i \\ n:\tilde{\mathbf{u}}_n &= \sum_{i=1}^{6} B_i \boldsymbol{\Psi}_i \quad , \quad \tilde{\boldsymbol{\sigma}}_n = \sum_{i=1}^{6} B_i \mathbf{F}^{-1}\mathbf{D}\boldsymbol{\Psi}_i \\ d:\tilde{\mathbf{u}}_d &= \sum_{i=1}^{6} B_i \boldsymbol{\Psi}_i e^{jk_{z,i}(d-d_p)} \quad , \quad \tilde{\boldsymbol{\sigma}}_d = \sum_{i=1}^{6} B_i \mathbf{F}^{-1}\mathbf{D}\boldsymbol{\Psi}_i e^{-jk_{z,i}(d_p-d)}. \end{aligned} \tag{14}$$

The equilibrium equations at the internal interface are:

$$\begin{bmatrix}\tilde{\mathbf{u}}_m\\ \tilde{\boldsymbol{\sigma}}_m\end{bmatrix}=\begin{bmatrix}\tilde{\mathbf{u}}_n\\ \tilde{\boldsymbol{\sigma}}_n\end{bmatrix}+\begin{bmatrix}\mathbf{0}\\ \tilde{\mathbf{p}}\end{bmatrix}\quad,\quad \tilde{\mathbf{u}}=\begin{bmatrix}\tilde{u}_x\\ \tilde{u}_y\\ \tilde{u}_z\end{bmatrix}\tilde{\boldsymbol{\sigma}}=\begin{bmatrix}\tilde{\sigma}_{xz}\\ \tilde{\sigma}_{yz}\\ \tilde{\sigma}_{zz}\end{bmatrix}. \tag{15}$$

The material of the two sublayers is the same, therefore the following derivations are possible:

$$\begin{aligned}&\underbrace{\begin{bmatrix}\boldsymbol{\Psi}_1 & \cdots & \boldsymbol{\Psi}_6\\ \mathbf{F}^{-1}\mathbf{D}\boldsymbol{\Psi}_1 & \cdots & \mathbf{F}^{-1}\mathbf{D}\boldsymbol{\Psi}_6\end{bmatrix}}_{\mathbf{M}}\begin{bmatrix}A_1\\ \vdots\\ A_6\end{bmatrix}=\underbrace{\begin{bmatrix}\boldsymbol{\Psi}_1 & \cdots & \boldsymbol{\Psi}_6\\ \mathbf{F}^{-1}\mathbf{D}\boldsymbol{\Psi}_1 & \cdots & \mathbf{F}^{-1}\mathbf{D}\boldsymbol{\Psi}_6\end{bmatrix}}_{\mathbf{M}}\begin{bmatrix}B_1\\ \vdots\\ B_6\end{bmatrix}+\underbrace{\begin{bmatrix}\mathbf{0}\\ \hat{\mathbf{p}}\end{bmatrix}}_{\mathbf{f}},\\ &\mathbf{MA}=\mathbf{MB}+\mathbf{f}\\ &\mathbf{A}=\mathbf{B}+\mathbf{M}^{-1}\mathbf{f}=\mathbf{A}_{\text{hom}}+\mathbf{A}_{part}\Rightarrow\mathbf{A}_{\text{hom}}=\mathbf{B}\ ,\ \mathbf{A}_{part}=\mathbf{M}^{-1}\mathbf{f}.\end{aligned} \tag{16}$$

At the original interfaces the following equations holds:

$$\begin{aligned}&\boldsymbol{\Theta}=\begin{bmatrix}\boldsymbol{\Theta}_u\\ \boldsymbol{\Theta}_d\end{bmatrix}=\begin{bmatrix}\boldsymbol{\Psi}_1 e^{-jk_{z,1}d/2}\ldots\boldsymbol{\Psi}_6 e^{-jk_{z,6}d/2}\\ \boldsymbol{\Psi}_1 e^{jk_{z,1}d/2}\ldots\boldsymbol{\Psi}_6 e^{jk_{z,6}d/2}\end{bmatrix},\ \Xi=\begin{bmatrix}\Xi_u\\ \Xi_d\end{bmatrix}=\begin{bmatrix}\mathbf{F}^{-1}\mathbf{D}\boldsymbol{\Theta}_u\\ \mathbf{F}^{-1}\mathbf{D}\boldsymbol{\Theta}_d\end{bmatrix},\ \mathbf{P}=\begin{bmatrix}\mathbf{P}_u\\ \mathbf{0}\end{bmatrix},\\ &\mathbf{P}_u=\left[\left[\mathbf{M}^{-1}\mathbf{f}\right]_1 e^{-jk_{z,1}z_p}\quad\cdots\quad\left[\mathbf{M}^{-1}\mathbf{f}\right]_6 e^{-jk_{z,6}z_p}\right]^T.\end{aligned} \tag{17}$$

The stresses are given by:

$$\tilde{\boldsymbol{\sigma}}_{part}=\Xi\mathbf{P}\ ,\ \tilde{\boldsymbol{\sigma}}_{\text{hom}}=\Xi\boldsymbol{\Theta}^{-1}\tilde{\mathbf{u}}_{\text{hom}}. \tag{18}$$

With this operation, the unknowns are mapped to the upper interface. This procedure is stable, as long as the load is not near to the lower boundary of the original layer. For this case, a mapping to the lower interface should help. Using Eq. 14, also a calculation of the stresses and displacement inside the layer is possible, but up to now, no useful results were gained. Therefore, the calculation of the internal stresses and displacements is done by splitting up the layer into two sublayers again.

3. BOUNDARY METHOD IN 2.5D

In the ongoing part of the text it is assumed that the structure is infinitely long and straight without a change in the cross-section. Therefore, the Fourier integral transformation is used with respect to the longitudinal axis *x*:

$$\breve{f}(k_x,y,z,\omega)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y,z,t)e^{-j(k_x x+\omega t)}dx\,dt. \tag{19}$$

The result is a spectral formulation that is named 2.5D.

The boundary element method in 2.5D can be derived using modified Plancherel's theorem:

$$\int_{-\infty}^{\infty}f(x)\cdot g(x)\,dx=\int_{-\infty}^{\infty}\breve{f}(-k_x)\cdot\breve{g}(k_x)\,dk_x. \tag{20}$$

In the spectral domain a coupling of the positive wavenumbers and the negative wavenumbers is given. Therefore, for the FEM a variation with respect to the negative wavenumbers is needed to derive a set of equations with unknowns at the positive wave number k_x.

In the boundary element method the single layer potential and the double layer potential are transformed using Plancherel's theorem.

$$\begin{aligned}&\mathbf{c}\,\hat{\mathbf{u}}(x_0=0,y_0,z_0,\omega)=\\ &\int_{-\infty}^{\infty}\int_{(\Gamma)}\hat{\mathbf{u}}^*(x,y-y_0,z,z_0,\omega)\,\hat{\mathbf{t}}(x,y,z,\omega)d\Gamma dx-\int_{-\infty}^{\infty}\int_{(\Gamma)}\hat{\mathbf{t}}^*(x,y-y_0,z,z_0,\omega)\,\hat{\mathbf{u}}(x,y,z,\omega)d\Gamma dx.\end{aligned} \tag{21}$$

The integral-free term is transformed using the inverse transform of the spectrum assuming that the source position is (x_0=0, y_0, z_0).

$$\begin{aligned}&c_o\,\breve{u}_o(k_x,y_0,z_0,\omega)=\\ &=\sum_{m=1}^{3}\int_{\Gamma}\breve{u}^*_m(-k_x,y-y_0,z,z_0,\omega)\,\breve{t}_m(k_x,y,z,\omega)d\Gamma-\sum_{m=1}^{3}\int_{\Gamma}\breve{t}^*_m(-k_x,y-y_0,z,z_0,\omega)\,\breve{u}_m(k_x,y,z,\omega)d\Gamma.\end{aligned} \tag{22}$$

Using the Green's function at $-k_x$ leads to a set of equations with unknowns that depend on the positive wave number k_x. **u** are the displacements and **t** the stresses acting on the boundary. The participation factor is c_0.

The elements in the transformed domain are line elements of length l with an angle of a between the local and global coordinates (Fig: 2)

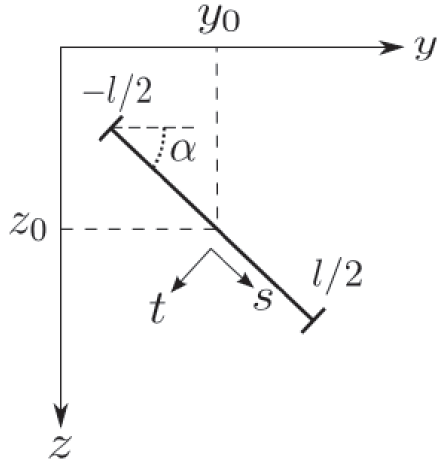

Figure 2 – Global coordinates *y, z* and local coordinates *s, t*.

The rotation of the stresses and displacements are done with:

$$\begin{bmatrix}\tilde{u}_1\\ \tilde{u}_2\\ \tilde{u}_3\end{bmatrix}=\begin{bmatrix}\tilde{u}_x\\ \tilde{u}_s\\ \tilde{u}_t\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos(\alpha) & -\sin(\alpha)\\ 0 & \sin(\alpha) & \cos(\alpha)\end{bmatrix}\begin{bmatrix}\tilde{u}_x\\ \tilde{u}_y\\ \tilde{u}_z\end{bmatrix} \tag{23}$$

and

$$\begin{bmatrix}\tilde{t}_1\\ \tilde{t}_2\\ \tilde{t}_3\end{bmatrix}=\begin{bmatrix}\tilde{\sigma}_{xt}\\ \tilde{\sigma}_{st}\\ \tilde{\sigma}_{tt}\end{bmatrix}=\begin{bmatrix}\tilde{\sigma}_{xy}\sin(\alpha)+\tilde{\sigma}_{xz}\cos(\alpha)\\ -\tfrac{1}{2}\left(\tilde{\sigma}_{zz}-\tilde{\sigma}_{yy}\right)\sin(2\alpha)+\tilde{\sigma}_{yz}\cos(2\alpha)\\ \tfrac{1}{2}\left(\tilde{\sigma}_{zz}+\tilde{\sigma}_{yy}\right)+\tfrac{1}{2}\left(\tilde{\sigma}_{zz}-\tilde{\sigma}_{yy}\right)\cos(2\alpha)+\tilde{\sigma}_{yz}\sin(2\alpha)\end{bmatrix}. \tag{24}$$

4. BOUNDARY ELEMENT INTEGRALS IN THE WAVENUMBER DOMAIN

The difficulty that is given now, is that the Greens' function is known in the domain (k_x, k_y, z, ω), but the boundary integrals are needed in the (k_x, y, z, ω) domain. The solution derived is to change the order of element integration and inverse Fourier transformation (5). The result for the single layer potential with constant shape functions is:

$$\begin{aligned} I_{SL} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\sum_{m=1}^{3}\sum_{i=1}^{6}\Big[\tilde{\mathbf{u}}_{i,m}\left(-k_x,k_y,\omega\right)\operatorname{sinc}\Big(\ell/2\Big(k_{z,i}\left(-k_x,k_y,\omega\right)\ \sin\alpha+k_y\cos\alpha\Big)\Big)\cdot \\ \cdot\breve{t}_{0,m}\left(k_x,\omega\right)e^{jk_{z,i}\left(-k_x,k_y,\omega\right)z_p}\ e^{jk_y\left(y_p-y_t\right)}\ dk_y\Big]. \end{aligned} \tag{25}$$

For the double layer potential, the following result is derived:

$$\begin{aligned} I_{DL} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\sum_{m=1}^{3}\sum_{i=1}^{6}\Big[\tilde{t}_{i,m}\left(-k_x,k_y,\omega\right)\operatorname{sinc}\Big(\ell/2\Big(k_{z,i}\left(-k_x,k_y,\omega\right)\ \sin\alpha+k_y\cos\alpha\Big)\Big)\cdot \\ \cdot\breve{u}_{0,m}\left(k_x,\omega\right)e^{jk_{z,i}\left(-k_x,k_y,\omega\right)z_p}\ e^{jk_y\left(y_p-y_t\right)}\ dk_y\Big]. \end{aligned} \tag{26}$$

The constant shape function leads to sinc functions in the transformed domain. This function attenuates the spectrum and makes a numerical evaluation of the spectra possible. If a linear shape function is used the derivative of the sinc function occurs.

5. FINITE ELEMENT METHOD IN 2.5D

The finite elements for the tunnel structure are derived in 2.5D and coupled with the boundary elements in 2.5D. Volume elements in 3D are reduced to plane elements in 2.5D. These elements are used for the tunnel shell and the superstructure. The rail is defined by a line element in 3D that is reduced to a point element in 2.5D.

6. TRANSITION TO 3D

The transition to 3D is done using the inverse Fourier integral transformation. This step is done by a Filon method (6) together with a quadrature for the demodulated spectrum. At the moment Clenshaw Curtis quadrature is used, because this algorithm reuses already calculated values from a prior step.

7. FIRST EXAMPLES

In a test calculation for the boundary element part without the FEM part, the dynamic deformations of the tunnel at a specific frequency were determined (Fig. 3 and Fig. 4).

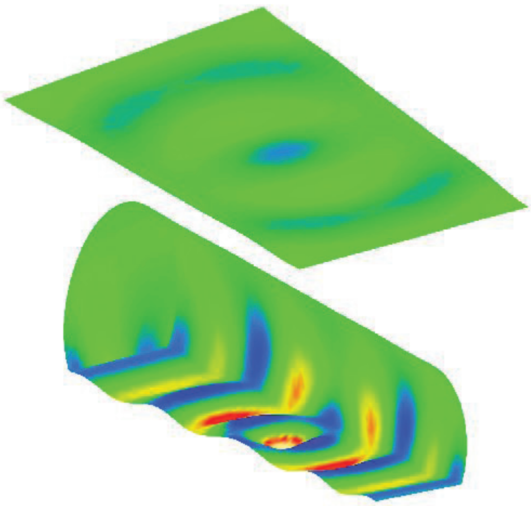

Figure 3 – Deformation of the tunnel shell and the free surface.

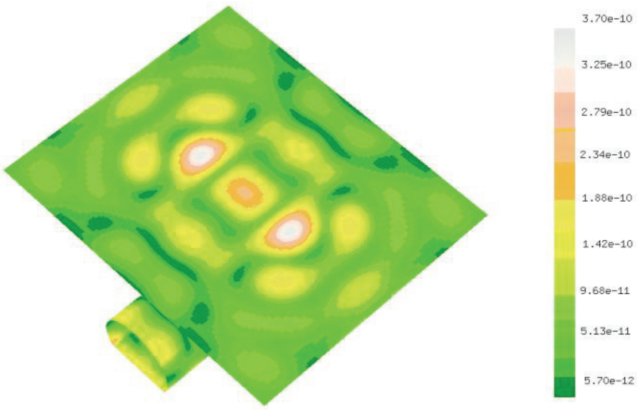

Figure 4 – Deformation of the tunnel shell and the free surface (top view).

8. CONCLUSIONS

A model that allows determining vibrations in a stratified medium with a tunnel structure inside is presented. The model solves the element integrals of the boundary integrals in the wavenumber domain, because in the wavenumber domain the Green's functions are given analytically. The inverse transformation is simplified, because sinc-functions and derivatives of the sinc-function attenuate the spectra.

The BEM part for the soil will be linked to the FEM part for the tunnel structure for the simulation of the whole system in the near future. The BEM part is solved using a new procedure and the FEM part will be solved using volume elements in 2.5D. This FEM part exists already in literature (7) and has to be adapted for the coupling with the BEM part.

ACKNOWLEDGEMENTS

The authors are grateful to the Austrian Academy of Sciences for supporting the project with the Innovation fund IF_2017_09 “Railway vibrations from tunnels”.

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