Evaluation of the Delany-Bazley-Miki model for the calculation of the sound absorption coefficient of aerogel granules

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Introduction

Aerogels are nanostructured, open porous solids, having a high porosity of 90 – 99.9 % [1]. Due to their low thermal conductivity of 0.005 – 0.1 W/(mK) they are found in applications in various industrial areas as in home building, pipelines, packaging, cryogenic tank insulation etc. Aerogels however have further additional application potentials in the aviation industry as sound absorbent material in the aircraft cabin wall for future aircraft designs since especially the granule forms of the aerogels have higher sound absorption properties than the aviation industry certified glass wool in the low frequency range, considering equivalent thermal isolation as well as equivalent material thickness (see Fig 1). It should be noted that, compared to the glass wool, the main disadvantage of the aerogels is presently their higher density for being used as an aircraft cabin wall isolation material.

In order to evaluate the sound absorption properties of materials, calculation models requiring certain input parameters, introduced in [2] and [3], need to be applied. For the aerogel granules measured data of the micro-geometry input parameters, as viscous and thermal characteristic lengths for instance, could not be found in the literature. In [4] a theoretical method has been considered to estimate the material parameters of the aerogel granules which are required for the Johnson-Champoux-Allard (JCA) model application. However, the calculated data are not confirmed by other references or different approaches. Delany and Bazley presented in 1969 an empirical model for the estimation of the characteristic impedance and the propagation coefficient of fibrous absorbent materials with a porosity near unity [5], which requires only one material parameter, the easily measurable flow resistivity. The characteristic impedance and the propagation coefficient estimated by the Delany-Bazley model can be used to predict the sound absorption coefficient of the applied materials. The estimation is based on simple power law relations whose parameters have been modified in 1989 by Miki [6] for the purpose of satisfying the impedance function positive real part property by fulfilling the Kramers–Kronig relations. Even though the Delany-Bazley-Miki model finds widespread application, however, since the estimation of the characteristic impedance and the propagation coefficient is based on simple power-law relations, it may not be expected that the Delany-Bazley approach provides reasonable data for all the porous fibrous and non-fibrous materials. Also, for the aerogel granules, the Delany-Bazley-Miki model original power law parameters don't result in accurate values for the sound absorption coefficient, as shown exemplarily in Fig. 2. Hence, in this paper a modification of the power-law parameters for the prediction of the sound absorption coefficient and its comparison with impedance tube measured data for aerogel granules of different particle sizes is discussed. It will be shown that for several aerogel granules particle sizes the adaptation of the Delany-Bazley power law relations parameters provides calculated data for the sound absorption coefficient that are in accordance with the impedance tube measurements, while for certain particle sizes the Delany-Bazley approach fails.

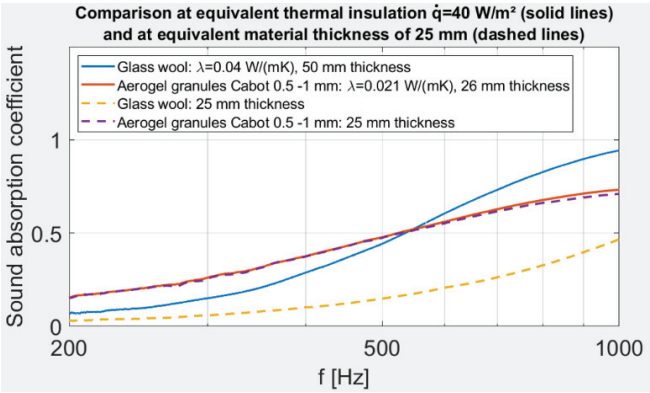

Figure 1: Comparison of sound absorption coefficients between aviation industry certified glass wool and aerogel granules of 0.5 – 1 mm particle size at equivalent heat transfer coefficient $\dot{q} = 40\ \mathrm{W/m^2}$ (solid lines) and at equivalent material thickness of 25 mm (dashed lines).

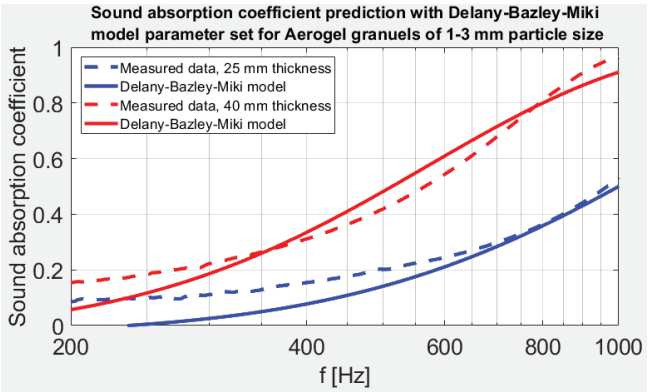

Figure 2: Comparison of impedance tube measured sound absorption coefficients and the Delany-Bazely-Miki model with original power law parameters calculated data for the aerogel granules 1 – 3 mm particle size.

Modification of the Delany-Bazley-Miki model

To adopt the Delany-Bazley-Miki model for the estimation of the characteristic impedance and the wave number of aerogel granules having different particle sizes, the parameters of the applied power-law relations need to be adjusted. The particle sizes of the used aerogel granules of the vendor CABOT are 10 µm – 4 mm, which have been fractionated by the German Aerospace Center (DLR) in particle sizes ranges 1 – 3 mm, 0.5 – 1 mm and 0.25 – 0.5 mm. It should be noted, however, that the different particles mixing ratio and their statistical distribution inside the mentioned granule fractions are unknown and have not been determined within the frame of this work. The input parameter of the Delany-Bazley-Miki model, the flow resistivity σ of the applied materials, is measured using the alternating flow method based on ISO 9053-2. The obtained data in kNs/m^4 are 16.6 for the 10 µm – 4 mm particle size range, 18 for the 1 – 3 mm particle size range, 115 for the 0.5 – 1 mm particle size range and amongst all mentioned the highest value of 186.9 for the 0.25 – 0.5 mm particle size range. For determining the parameters of the power-law relations, the characteristic impedance Z_c and the wave number k of the applied materials are measured with the 4-microphone-method (ASTM E2611-09) using an impedance tube in vertical position. The obtained data are the real and imaginary part of Z_c (R respectively X) and of k (α respectively β) as a function of the frequency over the flow resistivity f/σ, which are normalized with respect to air and plotted as curves of cluster of points, as shown in Fig. 3 a – d. As the Delany-Bazley approach the curves can be used to establish empirical relations based on the power-law functions

$$R = 1 + a(f/\sigma)^b, \tag{1}$$

$$X = c(f/\sigma)^d \tag{2}$$

respectively

$$\alpha = 1 + q(f/\sigma)^s \tag{3}$$

and

$$\beta = p(f/\sigma)^r \tag{4}$$

by determining the parameters $a, b,$ respectively q, s using a regression method. It should be noted that, as suggested by Miki [6], the following conditions defined in (5) and (6) for the parameters d, c, b, r and p are mandatory to be fulfilled for satisfying the Kramers–Kronig relations that connects the real and imaginary parts of any complex function - in this case Z_c and k:

$$d = b,\ c = -a\cot\left(\frac{(b+1)\pi}{2}\right),\ -1 < b < 0, \tag{5}$$

$$r = s,\ p = -q\tan\left(\frac{r\pi}{2}\right). \tag{6}$$

After determining the power-law relations parameters defined in (1) to (4) (see Fig. 3 a – d), the complex functions for Z_c and k are obtained:

$$Z_c = R + j\,X \tag{7}$$

$$k = \alpha + j\,\beta \tag{8}$$

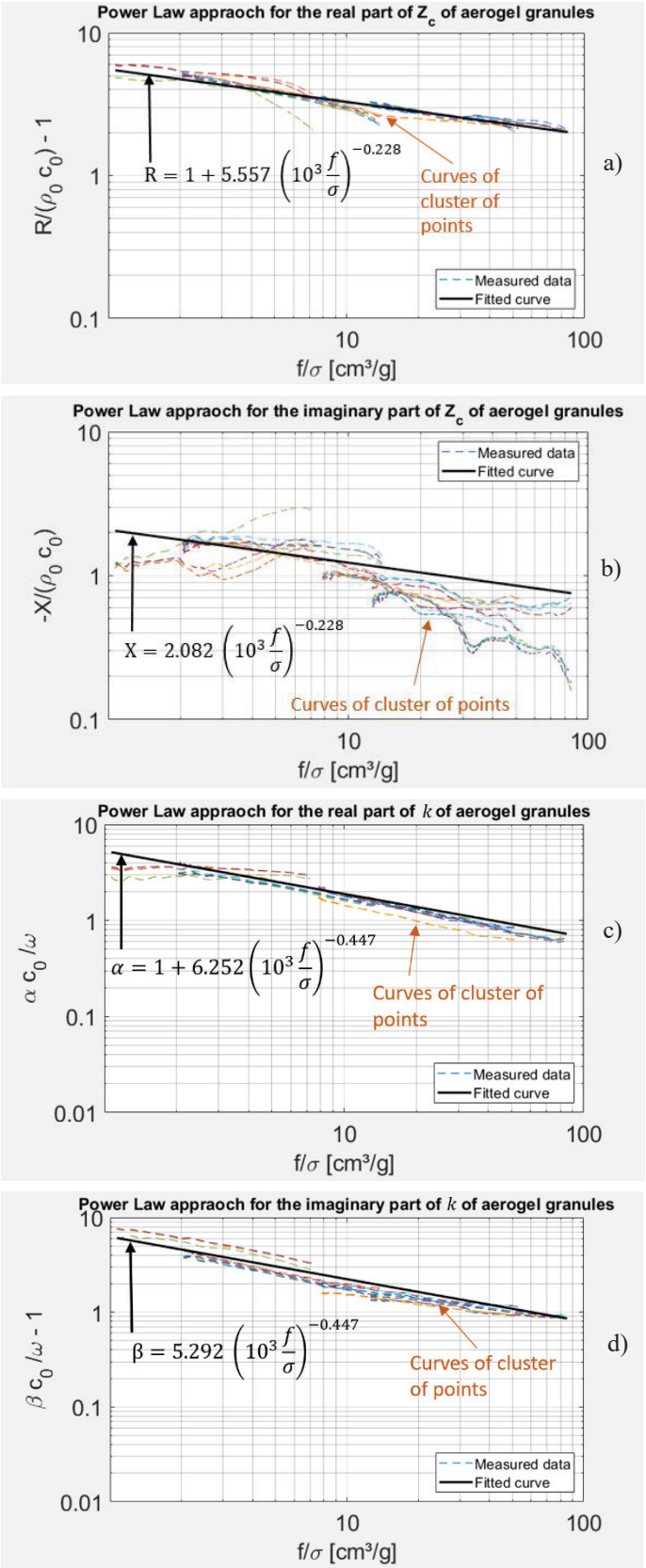

Figure 3: Approximation of a) the real part R and b) the imaginary part X of the characteristic impedance Z_c, respectively c) the real part α and d) the imaginary part β of the wave number k for the aerogel granules, based on a regression method, using curves of cluster of points, obtained by the impedance tube measurements. Unlike for the x-axis, in the given formulas f/σ is in MKS units.

By inserting expressions for R, X, α and β from equations (1), (2), (3), (4) in equations (7) and (8) the following relations for the characteristic impedance Z_c and the wave number k are obtained, whereas f/σ has to be inputted in MKS units:

$$Z_c = \rho_0 c_0 \left(1 + 5.557 \left(10^3 \frac{f}{\sigma}\right)^{-0.228} - j\, 2.082 \left(10^3 \frac{f}{\sigma}\right)^{-0.228}\right) \quad (9)$$

$$k = \frac{\omega}{c_0}\left(1 + 6.252\left(10^3 \frac{f}{\sigma}\right)^{-0.447} + j\, 5.292 \left(10^3 \frac{f}{\sigma}\right)^{-0.447}\right) \quad (10)$$

Using the material thickness d, now the surface impedance at normal incidence Z_W of the aerogel granules backed by an impervious rigid wall [2] can be calculated:

$$Z_W = -j\, Z_c \cot(k\, d) \quad (11)$$

Lastly the sound absorption coefficient α_{sac} for the normal incidence can be predicted using the relation

$$\alpha_{\text{sac}} = 1 - \left|\frac{Z_W - Z_0}{Z_W + Z_0}\right|^2, \quad (12)$$

where Z_0 = 413.3 Pa s/m is the impedance of the air.

Results discussion

A comparison of the results using the adapted Delaney-Bazley-Miki model computed sound absorption coefficients with impedance tube measured data is shown in Fig. 4. The modified model calculated data show a good agreement with the measured data in the frequency range 200 – 600 Hz for the aerogel granules of 10 µm – 4 mm particle sizes and the material thicknesses of 25 mm respectively 50 mm as presented in Fig. 4 a). In the higher than 600 Hz frequency range, the maximum deviation δ_{max} between the measured and calculated data is higher for both material sample thicknesses. The same behavior can be observed for the aerogel granules of 1 – 3 mm particle size and material thicknesses of 25 mm and 40 mm, as shown in Fig. 4 b): The agreement between measurement and calculated data in the frequency range lower than 600 Hz is higher than for the frequencies between 600 to 1000 Hz. For the aerogel granules of 0.5 to 1 mm particle size, the predicted data obtained by the modified model have also good accordance with the measured data. For this particle size and the material thickness of 30 mm, the average deviation δ in the frequency range higher than 600 Hz is even lower than for the frequencies 200 to 600 Hz, as illustrated in Fig. 4 c).

However, for the aerogel granules of the particle size 0.25 – 0.5 mm the modified model fails. The predicted data deviate significantly from the measured data in the entire frequency range (with exception of the curves intersections), as demonstrated for the material thicknesses of 25 mm and 40 mm in Figure 5. A possible reason might be associated with the relative high flow resistivity σ (186.9 kNs/m^4) measured

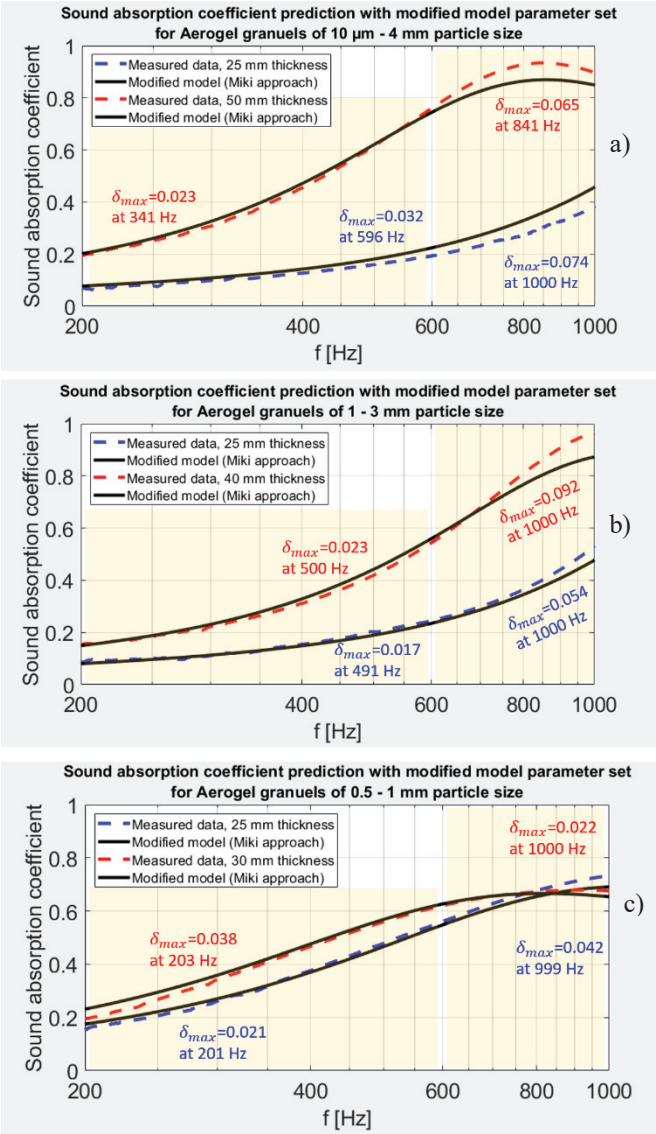

Figure 4: Comparison between measured and predicted data (modified model papameter) for the sound absorption coefficient of the aerogel granules with the particle sizes a) 10 µm – 4 mm b) 1 – 3 mm and c) 0.5 – 1 mm, with the corresponding maximum deviation δ_{max} for the frequency ranges 200 – 600 Hz and 600 – 1000 Hz.

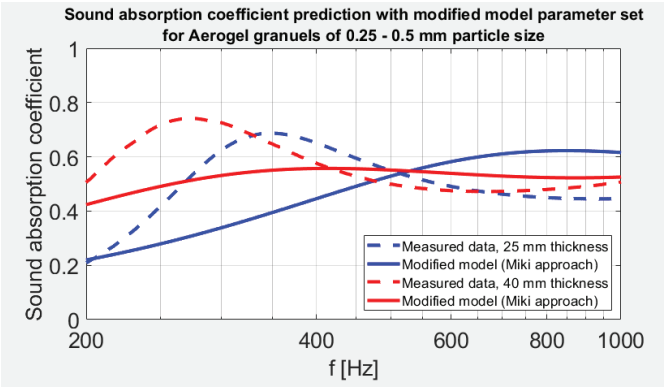

Figure 5: Comparison between measured and predicted data (modified model parameter) for the sound absorption coefficient of the aerogel granules of the particle sizes 0.25 – 0.5 mm. A huge deviation between measurement and prediction for the entire frequency range 200 – 1000 Hz (with exception of the curves intersections) can be observed.

for aerogel granules of the particle size 0.25 – 0.5 mm, which is significantly higher than those for all other investigated samples (see Table 1). Further clarification will be part of next research activities.

Conclusion

In this work the empirical parameters of the Delany-Bazley-Miki model have been modified to predict the sound absorption coefficient of aerogel granules of the vendor CABOT for samples of 4 different granules particle sizes 10 µm – 4 mm, 1 – 3 mm, 0.5 – 1 mm, and 0.25 – 0.5 mm. The modification is based on the determination of new parameters of the applied power-law relations, which is the core idea of the Delany-Bazley approach. For aerogel granules with particle sizes 10 µm – 4 mm, 1 – 3 mm, and 0.5 – 1 mm, the modified model provides reasonable values for the characteristic impedance Z_c and the wave number k. The calculated sound absorption coefficient for the mentioned aerogel granules particle sizes is in good agreement with the impedance tube measured data in the frequency range 200 to 1000 Hz having an average deviation $\delta \leq 0.035$. Furthermore, the modified model, as the original Delany-Bazley model, is rather easy for the application, since only one material property, the flow resistivity σ, is required as the input parameter, which can be measured using established methods. However, since the model takes only the flow resistivity into account and the estimation of the material's characteristic impedance and the wave number is based on simple power-law relations, it may not be expected that the model provides accurate data for all porous materials, as also noted by Allard and Atalla in [2] regarding the original Delany-Bazley model. For the aerogel granules of the particle size 0.25 – 0.5 mm, the modified model fails having huge discrepancy with the measured data. The maximum deviations for all the applied aerogel granules and the appropriate flow resistivities are summarized in the Table 1. As it can be noticed, the highest deviations between the calculated and measured data are exhibited by the aerogel granules with the highest flow resistivity, which might be associated with a currently unknown physical reason for the failure of the modified model for this particle size. In a detailed research, this unexpected behavior needs to be investigated. Also, the impact of the granules different particles mixing ratio and their statistical distribution on the modified model will be the topic of further works. Additionally, the evaluation of the modified model for the aerogel granules of the particle size lower than 0.25 mm and a consideration of the modified model for the frequency range higher than 1 kHz can be performed in a further research. Also the revision of the modified model for the aerogel granules of other vendors respectively for the granular form of other materials can be carried out in further investigations.

Applied aerogel granules particle size	σ [kNs/m^4]	Thick-ness [mm]	Max. deviation $\delta_{\max}$ of sound absorpt. coeff. (calculated/measured) (200 – 1000 Hz)
10 µm – 4 mm	16.6	25	0.074 at 1000 Hz
		50	0.065 at 841 Hz
1 – 3 mm	18	25	0.054 at 1000 Hz
		40	0.092 at 1000 Hz
0.5 – 1 mm	115	25	0.042 at 999 Hz
		30	0.038 at 203 Hz
0.25 – 0.5 mm	186.9	25	0.306 at 324 Hz
		40	0.233 at 262 Hz

Table 1: The maximum sound absorption coefficient deviations $\delta_{\max}$ between the calculated and measured data in the frequency range 200 – 1000 Hz for the applied aerogel granules and the appropriate flow resistivities.

Acknowledgments

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Literature

[1] L. Ratke, Aerogels: Structure, properties and applications, Conference Paper, 13th Meeting on Supercritical Fluids, ISASF 2011, October 2011

[2] J.F. Allard, N. Atalla, Propagation of Sound in Porous Media, Modelling Sound Absorbing Materials, Second Edition (Wiley, 2009).

[3] APMR: Acoustical Porous Material Recipes, Propagation models assuming a motionless skeleton: https://apmr.matelys.com/PropagationModels/MotionlessSkeleton/JohnsonChampouxAllardModel.html. [Accessed 04 August 2021].

[4] Y. Abawi, F. Langfeldt, H. Hoppen, W. Gleine, B. Milow, Application of the Johnson-Champoux-Allard model for the calculation of the sound absorption coefficient of aerogel granules based on inverse characterization for the determination of the granules parameters, DAGA 2020 Conference Paper, April 2020

[5] M.E.Delany, E.N.Bazley, Acoustical properties of fibrous absorbent materials, Applied Acoustics, Volume 3, Issue 2, April 1970

[6] Yasushi Miki, Acoustical properties of porous materials-Modification of Delany-Bazley models, J. Acoust. Soc. Jpn. (E) 11, 1 (1990)