Alternative Approaches to Generalized Sidelobe Canceler Beamformer

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Introduction

Beamforming is an important topic in array signal processing, that is the process of performing spatial filters to preserve the signal from directions of interest while suppressing interfering signals and noise arriving from other directions. A fixed beamformer is a beamformer whose coefficients is independent on the measurement signals, it normally uses to suppress stationary noise. On the other hand, an adaptive beamformer is a beamformer whose coefficients depend on or adapt to measurement signals, it is capable of suppressing interferences but it suffers from computationally intense and signal distortions.

J. Griffiths and W. Jim in [1] proposed the generalized sidelobe canceler (GSC) beamformer which is one of the most effective adaptive beamformings. In GSC, they modified the constrained problem in [2] to unconstrained problem by introducing a fixed beamforming at the upper path and a blocking matrix (BM) at the lower path. The lower path signals are filtered by the adaptive FIR filters before be subtracted from the upper path signal. This scheme could preserve the signal of interest (SOI) in the upper path while removing the noise in the lower path. Besides, the GSC is designed and processed in the time-domain, it is well-suited to broadband signals. However, the subtracting in GSC has some drawbacks, e.g., the GSC does not assure the noises/interferences in the lower path and in the upper path are almost identical, then substracting them does not suppress the noises/interferences effectively.

In this paper, we explain this issue from the beam pattern (BP) perspective. We regard the BM as the first order differential beamformings with a null at the look direction. Then, we analyze the difference between upper path's and lower path's BPs. In most of cases, the sidelobes of upper path BP is much different with the lower path's BP and the difference is varying over frequency. Then, the subtraction can not remove the sidelobes effectively. Furthermore, the BM generates multiple almost identical BPs at the lower path, that is somewhat redundant. To overcome those problems, we propose modified GSCs, that is, instead of using a BM in the lower path, we design a beamformer that has a null at look direction and some nulls at the nulls' position of the upper path BP. Some extra nulls at the lower path make the lower path BP is more similar to the sidelobes of the upper path BP. Besides the shape of BPs, the phase of BPs is also important that should be aware before subtracting the signals. For instance, the phase of the upper path and lower path BP are almost stable over the direction, we can use a fixed scale value in the lower path to correct the phase and amplitude of the subtracted signal. That is the case when the array layout is symmetric and the reference point is selected at the center of the array. On the other hand, if the phase difference is varying over the direction, we use an adaptive FIR filter at the lower path. Depending on the direction of interferences at an instance time, the FIR filter can scale the amplitude and adjust the phase of the lower path BP accordingly.

Generalized sidelobe canceler

Consider an array with M sensors, the GSC algorithm was proposed in the time-domain [1] where the signal of sensor m at the discrete time index n is defined by

$$x_m(n) = s_m(n) + n_m(n),\ m = 1, \ldots, M$$

where $s_m(n)$ and $n_m(n)$ are the SOI and noise, respectively. We can present the SOI at a sensor as the wave propagation from the source to the sensor $s_m(n) = \mathbf{a}_m * \mathbf{s}_0$ where $\mathbf{a}_m$ is the transfer function of the wave propagation, $\mathbf{s}_0$ is the source signal and '$*$' is the convolution operation.

Let $\mathbf{x}_A(n)$ be the time-alignment towards the look direction of $\mathbf{x}(n) = [x_1(n), \ldots, x_M(n)]^T$, normally $\mathbf{x}_A(n)$ is implemented by fraction delays of the measurement signals. Fig. 1 plots the overview of GSC.

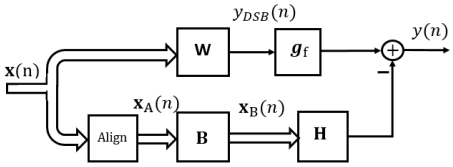


Figure 1: The block diagram of GSC.

In the lower path, the SOI portion $s_m(n)$ $m = 1, \ldots, M$ is aligned and suppressed via a BM:

$$\mathbf{x}_B(n) = \mathbf{B}\mathbf{x}_A(n)$$

where $\mathbf{x}_B(n)$ size of $M-1$ is the vector output of the BM, this signal vector should exclude the SOI.

The vector $\mathbf{x}_B$ is processed with adaptive FIR filters $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_{M-1}]^T$ and then be subtracted from the upper path's signal (e.g., the output of DSB) to get the noise-reduced signal, that is

$$y(n) = \sum_{i=0}^{L-1} y_{DSB}(n-i)\mathbf{g}_f[i] - \sum_{i=1}^{M-1}\sum_{j=0}^{L-1} \{\mathbf{x}_B(n-j)\}[i]\mathbf{h}_i[j] \quad (1)$$

where [.] indicates an element in a vector/matrix, $y_{DSB}(.)$ is the output of the DSB, $\mathbf{g}_f$ is a fixed FIR filter which ensures a specified gain and phase response for the output signal, L is the length of FIR filters.

The early paper of GSC [1] used an iterative procedure to adaptive updating $\mathbf{h}_i, \forall i = 1, \ldots, (M-1),\ k = 1, \ldots, L$ in the least-mean-square (LMS) sense

$$\{\mathbf{h}_i[k])\}_{n+1} = \{\mathbf{h}_i[k]\}_n + \mu_0 y(n)\{\mathbf{x}_B(n-k)\}[i] \quad (2)$$

where μ_0 is the normalized step size computed from a small factor β_0 by

$$\mu_0 = \frac{\beta_0}{\sum_{i=1}^{M-1}\sum_{j=0}^{L-1}(\{\mathbf{x}_B(n-j)\}[i])^2}. \quad (3)$$

From BP standpoint, we observe that one output of the BM $\mathbf{x}_B[m],\ m = 0, \ldots, M-2$ is equivalent to the output of differential beamforming, that is delayed and subtracted the signals. In case of a uniform linear array (ULA), its output contains $M-1$ almost identical first-order Cardioid BPs having null at look direction.

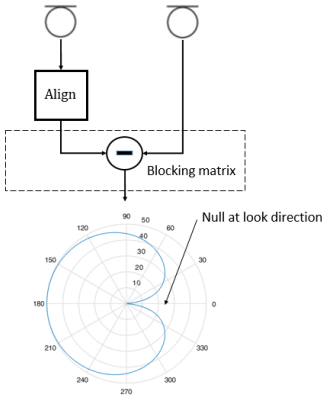


Figure 2: An output of the BM.

In the upper path, if a DSB is selected, then the gain of the BP obtains the maximum value at the look direction and less than that for other directions, e.g. Fig. 3 plots the DSB BP of ULA (endfire) with nine sensors (the inter-distance is 2.5 cm) at different frequencies.

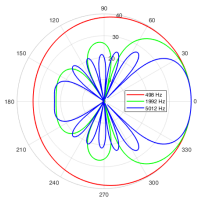


Figure 3: Example for the DSB's BPs.

It is clearly seen that the sidelobes of the upper path BPs are much different with the first-order Cardiod BPs in the lower path and the difference is varied over frequency. Therefore, the GSC needs to use adaptive filters that act as gain and phase controls for the first-order Cardiod BPs before subtracting them from upper path BP. Also, the GSC uses $M-1$ almost identical BPs in the lower path, that is somewhat redundant, because scaling and summing many identical BPs are equivalent to scaling a single BP.

Alternative approaches

The problem of subtracting different BPs in the GSC could be mitigated if we can design the lower path beamformer whose BP is almost identical to the sidelobes of upper path BP. That implementation is indeed feasible via the superdirectivity beamforming with multiple constraints (SDM) where the nulls position can be inserted arbitrarily [3].

At a narrow band, let $\mathbf{x}(\omega) = [x_1(\omega), \ldots, x_M(\omega)]^T$ be a measurement vector of the array at the rotation frequency ω , then the array response with beamforming is

$$y(\omega) = \mathbf{w}(\omega)^H \mathbf{x}(\omega) \quad (4)$$

where $\mathbf{w}(\omega) = [w_1(\omega), \ldots, w_M(\omega)]$ is the weight vector of the beamformer. The objective of SDM is to minimize the noise energy with subject to distortionless at the look direction and nulls at certain directions. Assume we want to design a BP having N nulls at $\theta_1, \ldots, \theta_N$ and the distortionless at θ_0, the solution of SDM is given by [3]

$$\mathbf{w}(\omega) = \frac{(\mathbf{\Gamma}(\omega) + \mu\mathbf{I})^{-1}\mathbf{D}(\omega)}{\mathbf{D}(\omega)^H(\mathbf{\Gamma}(\omega) + \mu\mathbf{I})^{-1}\mathbf{D}(\omega)}\mathbf{i}. \quad (5)$$

where $\mathbf{D}(\omega) = [\mathbf{d}(\omega, \theta_0), \mathbf{d}(\omega, \theta_1), \ldots, \mathbf{d}(\omega, \theta_N)]$ is the matrix size of $M \times (N+1)$ containing $N+1$ steering vectors, $\mathbf{\Gamma}(\omega)$, $\mathbf{I}$ are the correlation matrix of diffuse noise and white noise, respectively, μ is a number deciding the noise model and $\mathbf{i} = [1, 0, \ldots, 0]^T$ is a vector size of $N+1$. For conciseness, we omit ω in the formulas in the remainder of this section whenever possible.

To manipulate the shape of BP, instead of using the DSB in the upper path, we use a SDM in the upper path where the look direction is assured by the distortionless constraint, the sidelobe's shape and beamwidth are designed by nulls' positions. For instance, the different phase response of the BPs at the lower and upper paths are almost equal over the spatial directions except the mainlobe region of upper path BP, we could apply a fixed filter to align the phase and scale the amplitudes before subtracting them. That enables us to use fixed beamforming as an alternative approach to GSC. On the other hand, the different phase response of the BPs at the lower and upper paths is much varying over the spatial directions, we need to use the adaptive filter at the lower path, that is considered as a variant of GSC.

Fixed beamforming with suppressed sidelobes (FBSS)

At a narrow band, we design two beamformers, one is to construct the mainlobe with expected beamwidth (upper path BP) and the other is to replicate the sidelobes of the upper path BP (lower path BP). Then, we subtract the scale of lower path's signal from the upper path's signal to achieve a BP with suprressed sidelobes. Fig. 4 plots the block diagram of the fixed beamforming with suppressed sidelobes (FBSS).

On the upper path, the beamformer is designed by

$$\mathbf{w}_u = \frac{(\mathbf{\Gamma} + \mu\mathbf{I})^{-1}\mathbf{D}_u}{\mathbf{D}_u^H(\mathbf{\Gamma} + \mu\mathbf{I})^{-1}\mathbf{D}_u}\mathbf{i}_u \quad (6)$$

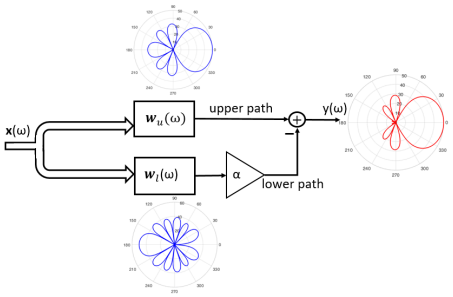


Figure 4: The block diagram of FBSS.

where $\mathbf{D}_u = [\mathbf{d}(\theta_0), \mathbf{d}(\theta_1), \ldots, \mathbf{d}(\theta_N)]$ is the matrix size of $M \times (N+1)$ containing $N+1$ steering vectors, and $\mathbf{i}_u = [1, 0, \ldots, 0]^T$ is a vector size of $N+1$.

On the lower path, the beamformer is designed by

$$\mathbf{w}_l = \frac{(\mathbf{\Gamma} + \mu\mathbf{I})^{-1}\mathbf{D}_l}{\mathbf{D}_l^H(\mathbf{\Gamma} + \mu\mathbf{I})^{-1}\mathbf{D}_l}\mathbf{i}_l \tag{7}$$

where $\mathbf{D}_l = [\mathbf{d}(\theta_m), \mathbf{d}(\theta_0), \mathbf{d}(\theta_1), \ldots, \mathbf{d}(\theta_N), \ldots, \mathbf{d}(\theta_{N+K})]$ is the matrix size of $M \times (N+K+2)$ containing $N+K+2$ steering vectors, θ_m is any direction that makes $\mathbf{w}_l \neq 0$ (e.g. θ_m is at the peak of sidelobes of upper path BP), $\theta_{N+1}, \ldots, \theta_{N+K}$ are the extra null directions in the upper path BP added by spatial aliasing effect when changing the frequency and $\mathbf{i}_l = [1, 0, 0, \ldots, 0]^T$ is a vector size of $N+K+2$.

The array response of the upper path beamforming at direction θ_m is

$$\alpha = \mathbf{w}_u^H \mathbf{d}(\theta_m) = \mathbf{i}_u^T \frac{\mathbf{D}_u^H(\mathbf{\Gamma} + \mu\mathbf{I})^{-1}\mathbf{d}(\theta_m)}{\mathbf{D}_u^H(\mathbf{\Gamma} + \mu\mathbf{I})^{-1}\mathbf{D}_u}. \tag{8}$$

If an array is symmetric to its reference point, we are easy to prove that the array response at the lower path and upper contain the real value only. Therefore, a FBSS is designed by

$$\mathbf{w} = \mathbf{w}_u - \alpha\mathbf{w}_l. \tag{9}$$

In a nutshell, we use (6), (7), (8) and (9) to design fixed beamforming for a symmetric array layout.

Adaptive beamforming with suppresed sidelobes (ABSS)

Analogous to the GSC, in this section, the proposed method is presented in the time-domain. In case of spatial phase response of beamforming is complicated (vary over direction), we need a robust approach that can update adaptively the phase and gain of lower path beamforming before subtracting it from the upper path beamforming, which is called an adaptive beamforming with suppressed sidelobes (ABSS). Depending on the interference directions, at a narrow band, the appropriate gain and phase shift for the lower path beamforming is updated via an adaptive FIR filter.

In Fig. 5, beamforming of the lower path and upper path is designed similarly to the FBSS, except that the coefficients are transformed to the time-domain as a set of

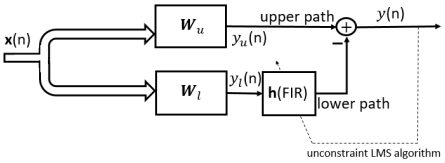


Figure 5: The block diagram of ABSS.

fixed spatial filters, i.e., $\mathbf{W}_l, \mathbf{W}_u$ are spatial filters' coefficients in time-domain for the lower path and upper path, respectively.

Beamforming at lower path is performed in the time-domain by spatial filtering

$$y_l(n) = \sum_{i=0}^{M-1}\sum_{j=0}^{L_1-1}\{\mathbf{x}(n-j)\}[i]\mathbf{W}_l[i,j] \tag{10}$$

where L_1 is the length of the spatial fitter of the lower path, $\mathbf{W}_l$ has a dimension of $M \times L_1$, computed from (7) for the frequencies of interest.

Beamforming at the upper path is performed in the time-domain by spatial filtering

$$y_u(n) = \sum_{i=0}^{M-1}\sum_{j=0}^{L_2-1}\{\mathbf{x}(n-j)\}[i]\mathbf{W}_u[i,j] \tag{11}$$

where L_2 is the length of the spatial fitter of the lower path (normally we set $L_1 = L_2$), $\mathbf{W}_u$ has a dimension of $M \times L_2$, computed from (6) for the frequencies of interest.

The final output of ABSS is given as

$$y(n) = y_u(n) - \sum_{i=0}^{L-1} y_l(n-i)\mathbf{h}[i] \tag{12}$$

where L is the length of the adaptive FIR filter $\mathbf{h}$. In Fig. 5, we adopt the normalized LMS algorithm for the adaptive FIR filter, that is

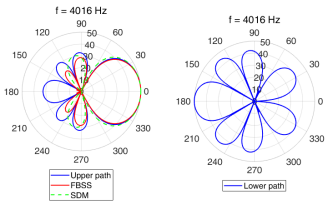
$$\{\mathbf{h}\}_{n+1} = \{\mathbf{h}\}_n + \frac{\beta}{\mathbf{b}^T\mathbf{b}}y(n)\mathbf{b}, \tag{13}$$

where β is the normalized step size with $0 < \beta < 1, \mathbf{b} = [y_l(n), \ldots, y_l(n-L+1)]^T$.

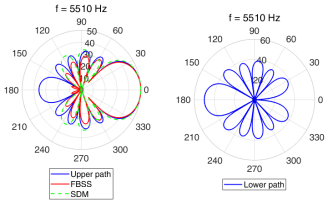
From the BP subtracting pointview, for the instantaneous interferences, the gain and phase in the lower path BP are updated accordingly to synchronize to the gain and phase of critical sidelobes of the upper path BP. Updating is performed via the sense of minimizing the output signal's energy.

Simulation for FBSS

We simulated broadband beamforming for a uniform linear array of $M = 7$ microphones with the inter-distance $d_H = 0.02$ m, the reference microphone is in the middle of the array. The upper path BP has a look direction at 0^o and fixed nulls at 70^o, 150^o. Then, the lower path BP has fixed nulls at 0^o, 70^o and 150^o, and we set $\theta_m = 180^o$. Due to spatial aliasing at high frequencies, there are some



(a) 4 kHz.



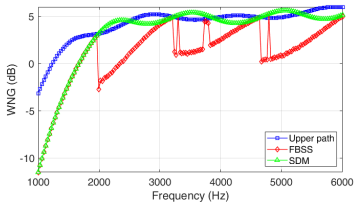
(b) 5.5 kHz

Figure 6: BPs at some frequencies: (left) upper path BP, FBSS and SDM, (right) lower path BP.

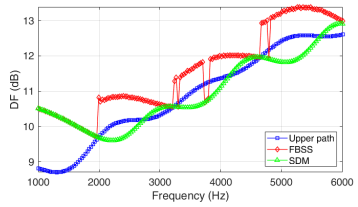
extra nulls at the upper path BP, we need to insert these nulls to the lower path BP as well. The noise model parameter μ in (5) is set 0.1 for all simulations.

In comparison, we also design SDM with a look direction at 0^o and fixed nulls at 70^o, 150^o and 180^o. In Fig. 6, we plotted the BPs at 4 kHz and 5.5 kHz, it is clear that the sidelobes of the upper path BP are suppressed for all frequencies, especially the sidelobe that contains $\theta_m = 180^o$ is almost completely suppressed.

The white noise gain (WNG) and directivity factor (DF) [4] of the upper path beamforming, FBSS and SDM are plotted in Fig. 7. It is clearly seen that, the FBSS improves the DF, while the WNG is still in the acceptable range.



(a) WNG.



(b) DF.

Figure 7: WNGs and DFs over frequency.

Simulation for ABSS

There are two interferences impinging on the array that have the variance σ_1^2, σ_2^2, then the signal-to-interference ratio (SIR) is defined as $\text{SIR} = 10\log_{10}\frac{\sigma_s^2}{\sigma_1^2+\sigma_2^2}$. where σ_s^2 is the variance of the SOI. To evaluate the performance of adaptive beamforming, we measure the signal's error (SE) between the SOI and beamforming output, that is $\text{SE} = E[(y(n) - s(n))^2]$ where $E[.]$ is the expectation value, $y(n)$ is the beamforming signal and $s(n)$ is the SOI. The SOI is simulated by summing a sinuous signal and two impulse responses and the SIR is changed from -30 dB to 10 dB, the SNR is set to 10 dB or 20 dB. We measure the SEs over SIR for the GSC, FBSS and ABSS with 25 Monte Carlo trials ($L = 35$). For beamformings inside ABSS, we use similar beamformings designed in the FBSS's simulation for a frequency range from 1 kHz to 6 kHz and applied an inverse discrete Fourier transfrom to obtain $\mathbf{W}_l, \mathbf{W}_u$ with $L_1 = L_2 = 128$, β is set 0.01. Fig.8 shows that the ABSS outperforms the GSC in terms of preserving the SOI and suppressing the interferences.

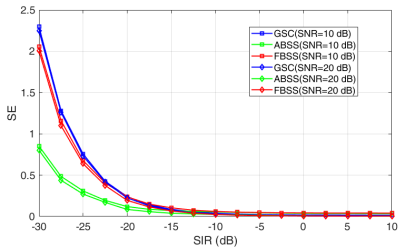


Figure 8: SEs over SIR for the ABSS, FBSS and GSC.

Conclusion

In this paper, we have proposed two alternative approaches to the GSC beamformer: One is for fixed beamforming, another is for adaptive beamforming. The proposed fixed beamforming applies for a symmetric array layout and the reference point must be at the center of the array. For the proposed adaptive beamforming, it can apply for an arbitrary array layout. This proposed method outperforms the GSC in terms of interference suppression and preserving the signal of interest. Moreover, the computational complexity of this method is less than that of the GSC because the proposed method uses one adaptive FIR filter.

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