Quantifying the coupling strength of adhesively bonded materials by investigating mode repulsion regions

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Abstract

In the field of non-destructive testing, Lamb waves are often used for material characterisation. The increasing computational capabilities further enable complex and detailed simulations to predict the material behaviour in reality. Since adhesive bonding of different materials is a widely used method in modern applications, a reliable measurement system to determine the quality of these adhesive bonds is needed. Investigations of the dispersive behaviour of acoustic waves in adhesively bonded multi-layered waveguides show mode repulsions in the dispersion diagrams in regions where the modes of the single materials would otherwise intersect. In previous works, changes of the distance between those modes with respect to the bonding quality are observed. The experimental data for this investigation is generated using pulsed laser radiation to excite broadband acoustic waves in plate like specimens which are detected by a piezoelectric ultrasonic transducer. Numerical data is generated using simulations via a semi-analytical finite element method. Using a combination of experimental and numerical data, the present work introduces an approach to determine a parameter which indicates the bonding quality in relation to an ideal material coupling.

Dispersion in plate-shaped acoustic waveguides

Under the assumption of a plate-shaped acoustic waveguide in vacuum the dispersion relation is given by the Rayleigh-Lamb equation:

$$\frac{\tan\left(qt/2\right)}{\tan\left(pt/2\right)} = -\left(\frac{4k^2pq}{\left(q^2-k^2\right)^2}\right)^{\pm 1} \tag{1}$$

with

$$p^2 = \frac{\omega^2}{c_\mathrm{L}^2} - k^2 \quad \text{and} \quad q^2 = \frac{\omega^2}{c_\mathrm{T}^2} - k^2 \tag{2}$$

and the plates thickness t, the longitudinal wave velocity c_L and the transversal wave velocity c_T. It contains material dependent parameters on the one hand and the angular frequency ω and the angular wavenumber k on the other hand [1]. Therefore, if the material parameters are known, equation (1) can be used to determine the angular wavenumbers for given angular frequencies (or vice versa) to determine the plates dispersive guided wave modes. As the dispersive behaviour depends on the material parameters (see equation (1) and equation (2)) the Rayleigh-Lamb equation can be used as a forward model in an inverse procedure to determine the materials bulk sound velocities [2]. Since there exists no analytical solution for equation (1), numerical root finding or semi-analytic methods have to be used [3, 4]. Semi-analytical methods provide the advantage of easily extendable simulation models, as it is needed e.g. for multi-layered systems [5].

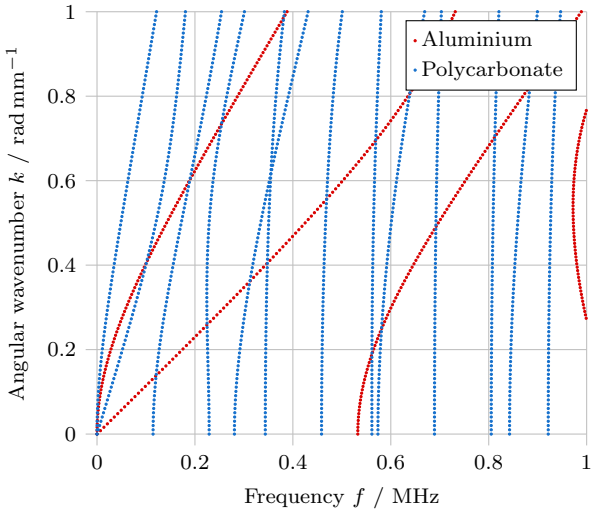

Figure 1: Dispersion diagram of a 3 mm tick aluminium plate and a 4 mm thick polycarbonate plate.

Table 1: Material parameters used for simulations in the presented investigation.

Material	Long. wave vel. c_L	Trans. wave vel. c_T
Aluminium	6384.7 m s^{-1}	3163.4 m s^{-1}
Polycarbonate	2216.7 m s^{-1}	904.2 m s^{-1}

Figure 1 graphically shows the solutions of the Rayleigh-Lamb equation for a single aluminium plate ($t = 3$ mm) as well as for a single polycarbonate plate ($t = 4$ mm). The visualised dispersion diagrams are calculated with a semi-analytical finite element method using the material parameters as provided in Table 1. These show the modes capable of propagation inside the specific material for a given angular wavenumber k and frequency f. Compared to aluminium, the bulk velocities of polycarbonate are lower which results in fewer Lamb modes in the same frequency range.

For a simulative investigation of the dispersive be-

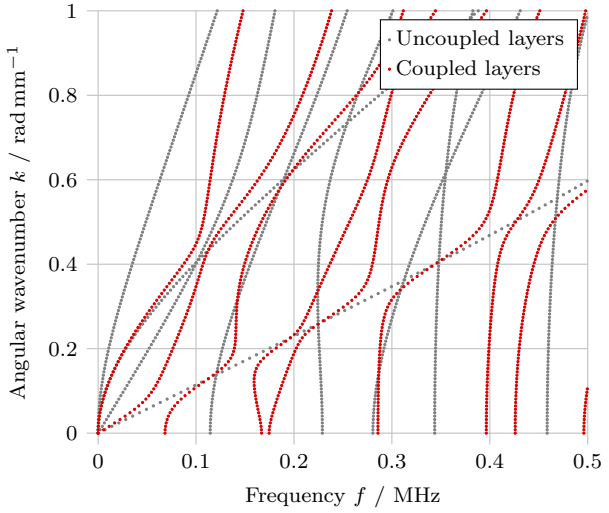

Figure 2: Dispersion diagram of a multi-layered system (red) consisting of a 3 mm thick aluminium plate and a 4 mm thick polycarbonate plate overlaid with the dispersion diagrams of the uncoupled single layers (grey).

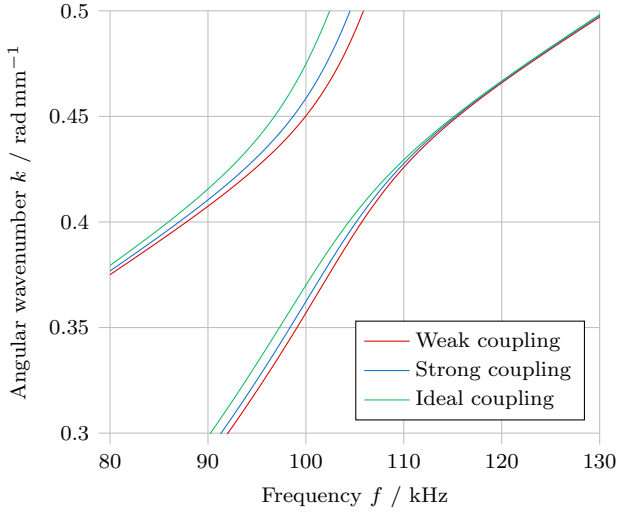

Figure 3: Visualisation of the influence of different coupling strengths between aluminium and polaycarbonate on the mode repulsion region between the first and second mode.

haviour of a multi-layered system the two material models used for the uncoupled simulations (Figure 1) are combined [5]. The result of such a simulation using the same parameters as for the uncoupled material models is depicted in Figure 2.

The dispersive behaviour is clearly influenced by the two layer materials, but a very prominent feature is the absence of mode crossings. While uncoupled, the layer materials can oscillate independently, this is no longer possible in multi-layered systems and leads to an effect called *mode repulsion* in the vicinity of theoretical crossings of the single layer modes. The phenomenon is clearly visible in Figure 2 at $f = 0.1\,\mathrm{MHz}$ and $k = 0.1\,\mathrm{rad\,mm^{-1}}$. Überall et al. [6] analysed a similar effect in phase velocity dispersion curves of two uncoupled fluid layers in comparison to the curves of a single solid layer and used this effect to analyse the coupling of longitudinal and transversal waves in the solid layer. Following this idea, the area around the crossing points of the uncoupled layers (refereed to as *mode repulsion regions*) will be investigated to determine the coupling strength in multi-layered systems as depicted in Figure 2. A previous approach by Lugovtsova et al. [7] models a non-ideal coupling of two materials by inserting a thin layer model between the two material models. Variation of this layer's stiffness indicates a change in the mode repulsion regions in the phase velocity dispersion curves with respect to the material coupling.

Following that same approach, the effect of different coupling strengths between an aluminium and a polycarbonate plate can be simulated by inserting a thin layer into the multi-layered model used for the simulation of Figure 2. By varying the stiffness of this layer a weak, strong and ideal coupling can be simulated. Figure 3 visualises the effect of these couplings on the repulsion region between the first and second mode of the aluminium polycarbonate system. As expected, the simulations indicate

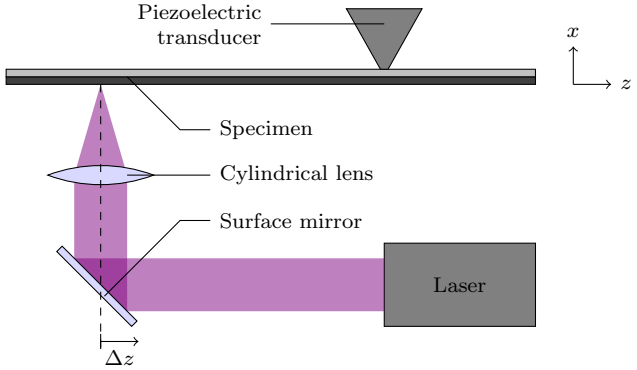

Figure 4: Schematic of the measurement setup for the broadband excitation and detection of acoustic plate waves.

an increasing distance between the two mode curves with increasing layer stiffness (and therefore increasing coupling strength).

Multilayer sample

To investigate changes in the dispersive behaviour around mode repulsion regions experimentally a real multilayer system is created. Thus, a 3 mm thick aluminium plate is adhesively bonded to a 4 mm thick polycarbonate plate using an epoxy adhesive (EA9483, *LOCTITE*). To be able to compare measurements of two different coupling strengths a first measurement is performed after 48 h of adhesive curing at room temperature (approximately 22 °C). At this point the adhesive has formed a significant bond between the two materials, but has not yet reached its maximum strength. This is indicated by an optically opaque epoxy layer. The measurement performed at this state will be referred to as partially cured. After additional 72 h the adhesive layer has fully cured as indicated by the optically fully transparent layer and a second measurement is performed.

Experimental setup

The measurements for the investigation at hand were performed using an experimental setup as depicted in

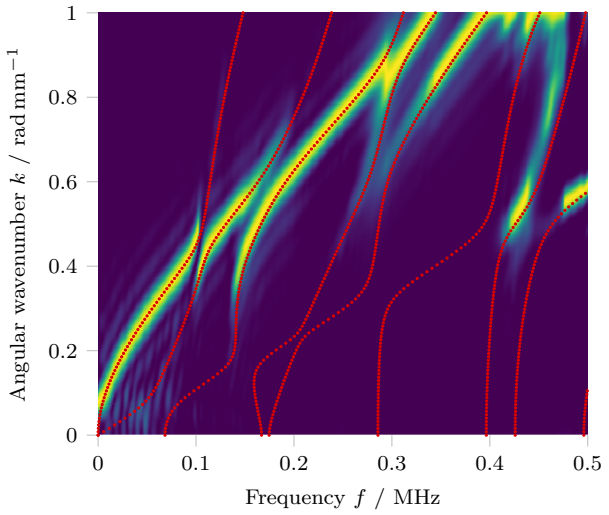

Figure 5: Processed measurement data (image) of a fully cured aluminium-polycarbonate sample overlaid by simulated ideally coupled dispersion diagram of such a multi-layered system.

Figure 4. Broadband plate waves are excited in plate-like specimens using a pulsed nitrogen laser (MNL 103-PD$^{\text{High Power}}$, *LTB Lasertechnik Berlin GmbH*). Using a surface mirror and a cylindrical lens the ultraviolet laser radiation is focused on a thin line ($\approx 0.1\,\text{mm}$) on the specimens surface. The optical energy density is sufficiently low to cause a purely thermoelastic excitation [8] without inflicting permanent damage to the sample. Due to the thin line focus the resulting mechanical waves have approximately planar wave fronts with high spatial bandwidth. The sufficiently short (3 ns) radiation pulses excite acoustical waves with a high temporal bandwidth. The propagating waves are detected using a purpose-built transducer with a strip-shaped piezoceramic (PIC255, *PI Ceramic*) as the active element. Due to a sufficient amount of damping mass in the transducer's casing the capability of broadband detection is ensured [2]. The sensor is placed on the specimens surface with the active element parallel to the assumed planar excited wavefronts. The optical components of the setup are mounted on a linear actuator allowing for a variation of the point of excitation on the specimens surface. The resulting measurement data therefore contains information in spatio-temporal domain.

Applying a two-dimensional Fourier transform to the measurement data matrix leads to data with information in the wavenumber-frequency domain. The absolute value of this transformed matrix has high signal values where propagating modes are detected, thus the depiction is visually similar to dispersion diagrams of the respective waveguides [9]. To enhance the quality of those images further data processing like gamma correction and column-wise normalisation can be applied.

Figure 5 shows processed measurement data of the fully cured aluminium-polycarbonate sample. The detected modes are clearly visible as independent yellow ridges. A subset of mode repulsion regions can be identified, which is further investigated. The measurement data is compared to the simulation results of an ideally coupled aluminium-polycarbonate system. Measurement data and simulation are generally in good agreement. The slight off-set can be explained by the neglection of an adhesive layer in the simulation. The adhesive layer of the specimen has an average thickness of 0.1 mm and is therefore significantly smaller than the smallest expected wavelength $\lambda_{\text{min}} = 2\pi/k_{\text{max}} = 6.28\,\text{mm}$ in the observed range, which enables the modelling as a negligible thin interface layer.

Modal distance

Since the target of this investigation is to quantify the coupling strength between two materials via properties of their mode repulsion regions, one has to define an observable parameter. Because of the work of Lugovtsova et al. [7] and Figure 3 it is known that the distance between the two mode in those regions depends on the coupling strength. A possible one dimensional approach for such a quantitate parameter could be the definition of a reference point, e.g. the theoretical crossing point of the uncoupled mode curves, and than determine the distance between that point and the mode curves in the frequency or angular wavenumber domain. But since the simulations have shown that the change in distance depends on both dimensions it makes sense to use a two dimensional approach like the Euclidean distance:

$$d = \sqrt{(f_1 - f_2)^2 + (k_1 - k_2)^2} \tag{3}$$

with f_n and k_n being the frequency and angular wavenumber of the two mode curves, where d becomes minimal. This form of distance calculation does not consider differences of magnitude in the two different dimensions. But since both dimensions differ significantly in their contributions, the influence of the angular wavenumber domain would be negligible. Therefore an average-proportional normalisation is used to determine a normalised modal distance d_{norm}:

$$d_{\text{norm}} = \sqrt{\left(\frac{f_1 - f_2}{f_1 + f_2}\right)^2 + \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2} \tag{4}$$

Referencing both domains to their own contributions accounts for any differences in magnitudes.

To enable a comparison of the modal distance metric between different materials and samples it is advantageous to use a relative parameter. Since the simulations of an ideal coupled systems have proven to be a fitting comparison for measurements, they can be used in this scenario as well. Therefore the relative modal distance will be calculated by dividing the normalised modal distance of a measurement $d_{\text{norm,meas}}$ by the normalised modal distance of the ideal coupled simulation $d_{\text{norm,sim}}$:

$$\tilde{d}_{\text{norm}} = \frac{d_{\text{norm,meas}}}{d_{\text{norm,sim}}} \tag{5}$$

Figure 6 visualises the normalised modal distances d_{norm} for the two stages of adhesive curing of the investigated

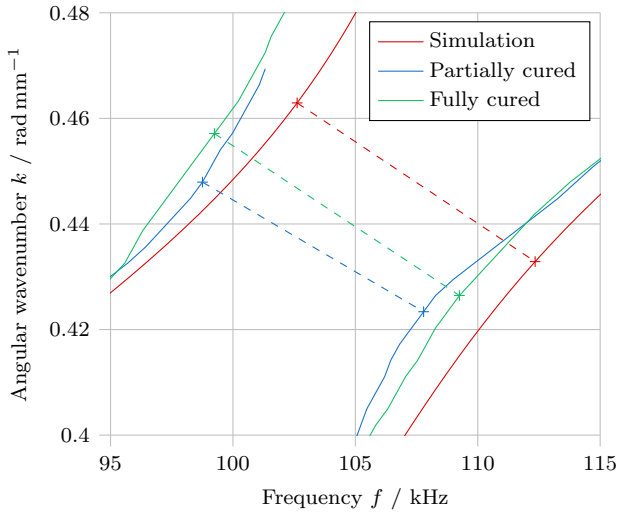

Figure 6: Minimal modal distances d_{norm} in the mode repulsion region between the first two modes of the adhesive bonded aluminium-polycarbonate sample at different stage of adhesive curing.

multi-layered sample. The blue and green curves are extracted from measurements using a peak detecting algorithm. Because of the incompletely cured state during the measurement for the blue curves, modes can only be identified over a smaller frequency/angular wavenumber range (see [5]). The red curves represent the simulated modes of the ideal coupled system. The dashed lines in Figure 6 visualise the minimal distance, and therefore the normalised modal distance d_{norm} between the respective curves. Using the simulated distance as a reference leads to relative modal distances of $\tilde{d}_{\mathrm{norm},0} = 0.92$ for the partially cured state and $\tilde{d}_{\mathrm{norm},1} = 1.05$ for the fully cured state. This indicates the same trend as expected from the simulations: With increasing coupling strength between the two materials the modal distance in this repulsion region increases as well. The deviations in the position of the repulsion region between measurement and simulation as well as the seemingly unphysical result $\tilde{d}_{\mathrm{norm},1} > 1$ are due to the assumptions made for the simulation model (ideal coupling and an adhesive layer with zero thickness).

Conclusion

A multi-layered plate-shaped specimen consisting of an aluminium plate bonded to a polycarbonate plate via an epoxy adhesive is investigated to determine changes in its dispersive behaviour during adhesive curing. The results indicate the possibility of deriving a quantitative measure for the coupling strength between the specimens layer materials by determining the minimal distance between two mode curves in regions where the mode curves of the uncoupled materials would have crossing points. By determining this modal distance in actual measurements and comparing them to simulations, previous findings are confirmed and a quantitative measure is provided. Further investigations will aim to derive a more detailed understanding of the connection between modal distance and the degree of coupling between the layer materials. Additionally, the authors intend to improve the measurement data processing further to get more detailed information about the specimens dispersive behaviour. In the long term, the complexity of the procedure will be reduced to enable usage in industrial applications.

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